TM CLASS CAVITY DESIGN

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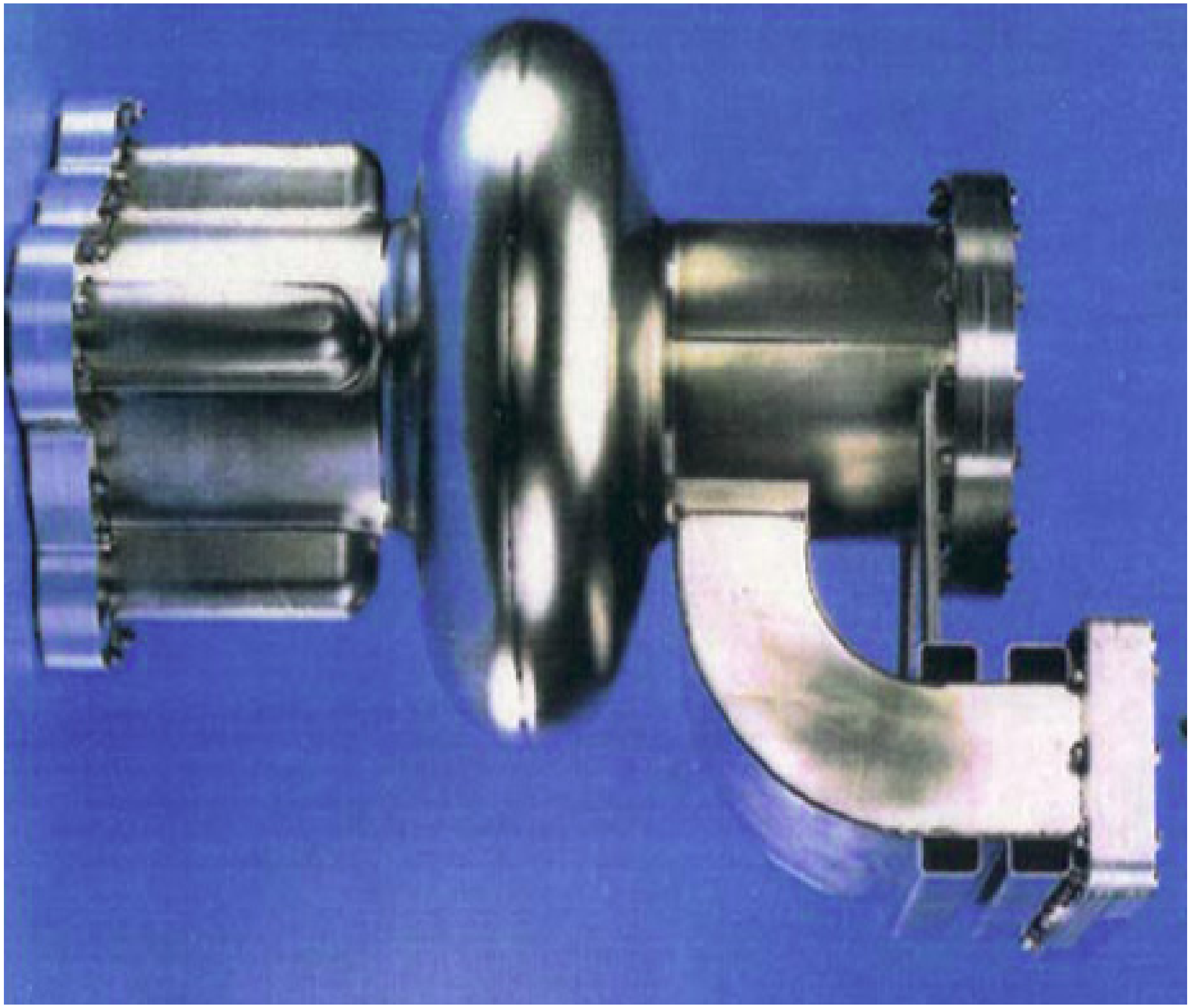
500 MHz, Single-cell



350 MHz, 4-cell, Nb on Cu

1500 MHz, 5-cell

1300 MHz 9-cell

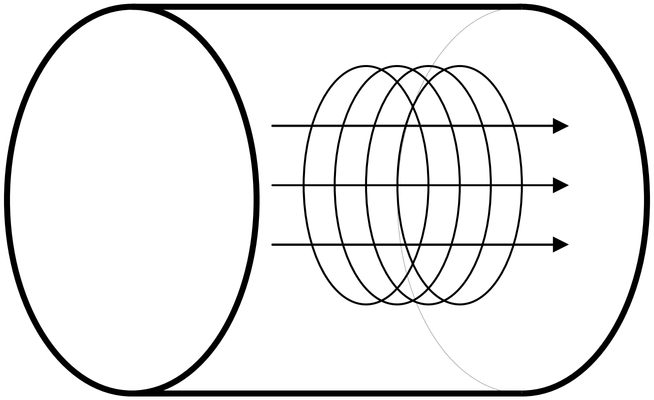
Pill Box Cavity

Hollow right cylindrical enclosure

Operated in the TM_{010} mode $\quad H_z = 0$

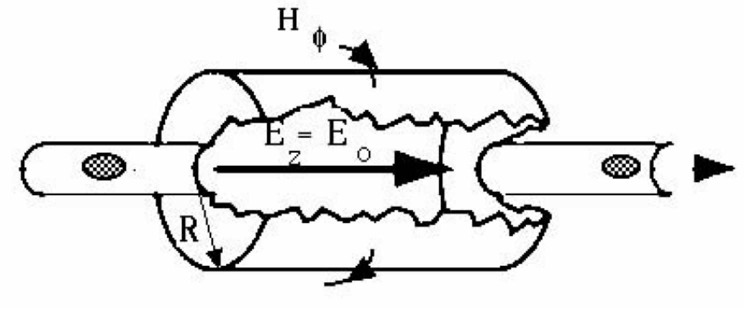
$$\frac{\partial^2 E_z}{\partial^2 r} + \frac{1}{r}\frac{\partial E_z}{\partial r} = \frac{1}{c^2}\frac{\partial^2 E_z}{\partial^2 t} \qquad \omega_0 = \frac{2.405c}{R}$$

TM_{010} mode



$$E_z(r,z,t) = E_0 J_0\left(2.405\frac{r}{R}\right)e^{-i\omega_0 t}$$

$$H_\varphi(r,z,t) = -i\frac{E_0}{\mu_0 c}J_1\left(2.405\frac{r}{R}\right)e^{-i\omega_0 t}$$



Modes in Pill Box Cavity

- TM_{010}
  - Electric field is purely longitudinal
  - Electric and magnetic fields have no angular dependence
  - Frequency depends only on radius, independent on length
- TM_{0mn}
  - Monopoles modes that can couple to the beam and exchange energy
- TM_{1mn}
  - Dipole modes that can deflect the beam
- TE modes
  - No longitudinal E field
  - Cannot couple to the beam

TM$_{lmn}$ Modes in a Pill Box Cavity

$$\frac{E_r}{E_0} = -\frac{n\pi}{x_{lm}}\frac{R}{L}J_l'\left(x_{lm}\frac{r}{R}\right)\sin\left(n\pi\frac{z}{L}\right)\cos l\varphi$$

$$\frac{E_\varphi}{E_0} = \frac{ln\pi}{x_{lm}^2}\frac{R^2}{rL}J_l\left(x_{lm}\frac{r}{R}\right)\sin\left(n\pi\frac{z}{L}\right)\sin l\varphi$$

$$\frac{E_z}{E_0} = J_l\left(x_{lm}\frac{r}{R}\right)\sin\left(n\pi\frac{z}{L}\right)\cos l\varphi$$

$$\omega_{lmn} = c\sqrt{\left(\frac{x_{lm}}{R}\right)^2 + \left(\frac{\pi n}{L}\right)^2}$$

$$\frac{H_r}{E_0} = -i\omega\varepsilon\frac{l}{x_{lm}^2}\frac{R^2}{r}J_l\left(x_{lm}\frac{r}{R}\right)\cos\left(n\pi\frac{z}{L}\right)\sin l\varphi$$

x_{lm} is the mth root of $J_l(x)$

$$\frac{H_\varphi}{E_0} = -i\omega\varepsilon\frac{R}{x_{lm}}J_l'\left(x_{lm}\frac{r}{R}\right)\cos\left(n\pi\frac{z}{L}\right)\cos l\varphi$$

$$\frac{H_z}{E_0} = 0$$

TM$_{010}$ Mode in a Pill Box Cavity

$$E_r = E_\varphi = 0 \qquad E_z = E_0 J_0\left(x_{01}\frac{r}{R}\right)$$

$$H_r = H_z = 0 \qquad H_\varphi = -i\omega\varepsilon E_0 \frac{R}{x_{01}} J_1\left(x_{01}\frac{r}{R}\right)$$

$$\omega = x_{01}\frac{c}{R} \qquad x_{01} = 2.405$$

$$R = \frac{x_{01}}{2\pi}\lambda = 0.383\lambda$$

TM$_{010}$ Mode in a Pill Box Cavity

Energy content

$$U = \varepsilon_0 E_0^2 \frac{\pi}{2} J_1^2(x_{01}) L R^2$$

Power dissipation

$$P = E_0^2 \frac{R_s}{\eta^2} \pi J_1^2(x_{01})(R+L)R$$

$$x_{01} = 2.40483$$

$$J_1(x_{01}) = 0.51915$$

Geometrical factor

$$G = \eta \frac{x_{01}}{2} \frac{L}{(R+L)}$$

TM010 Mode in a Pill Box Cavity

Energy Gain

$$\Delta W = E_0 \frac{\lambda}{\pi} \sin \frac{\pi L}{\lambda}$$

Gradient

$$E_{acc} = \frac{\Delta W}{\lambda / 2} = E_0 \frac{2}{\pi} \sin \frac{\pi L}{\lambda}$$

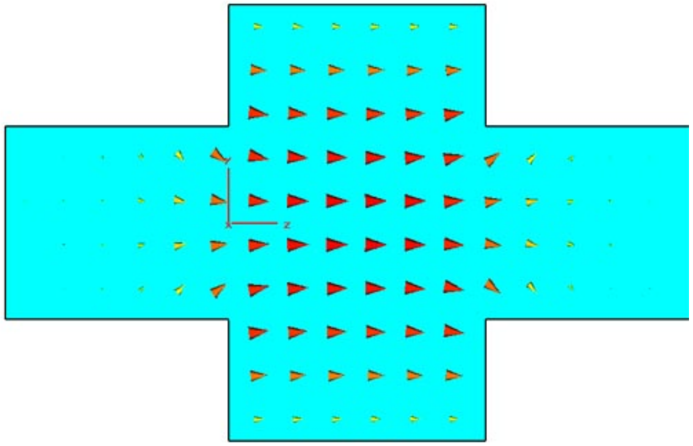
Shunt impedance

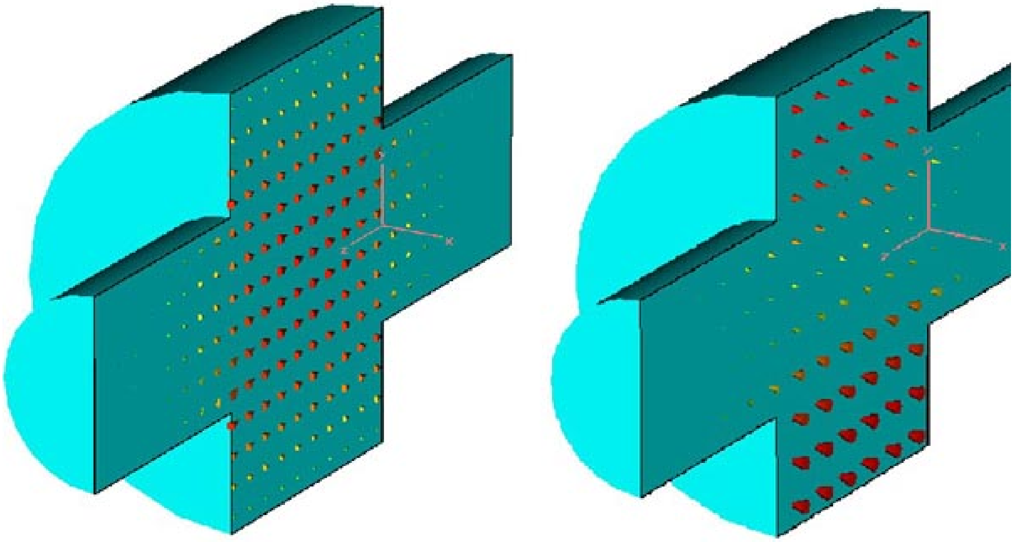
$$R_{sh} = \frac{\eta^2}{R_s} \frac{1}{\pi^3 J_1^2(x_{01})} \frac{\lambda^2}{R(R+L)} \sin^2\left(\frac{\pi L}{\lambda}\right)$$

Real Cavities

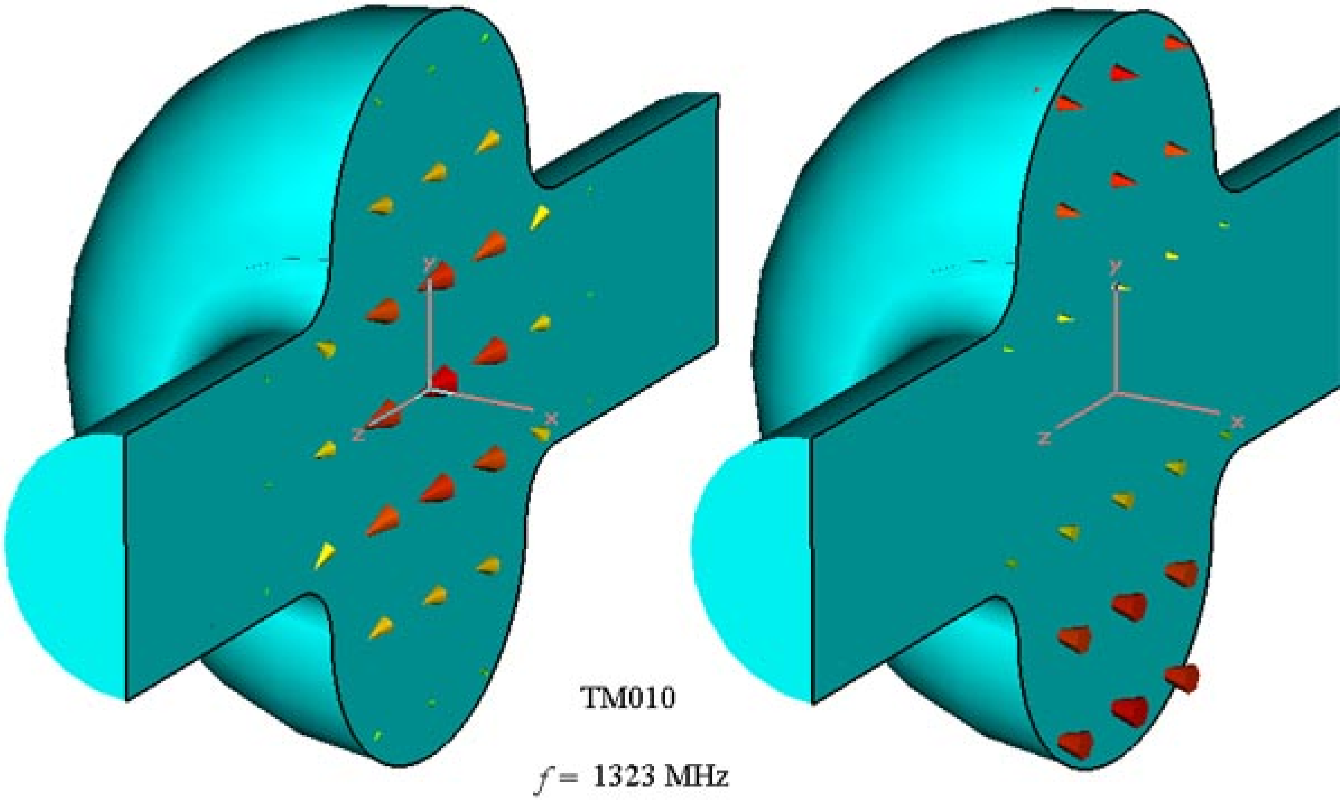
Beam tubes reduce the electric field on axis

- Gradient decreases
- Peak fields increase
- R/Q decreases



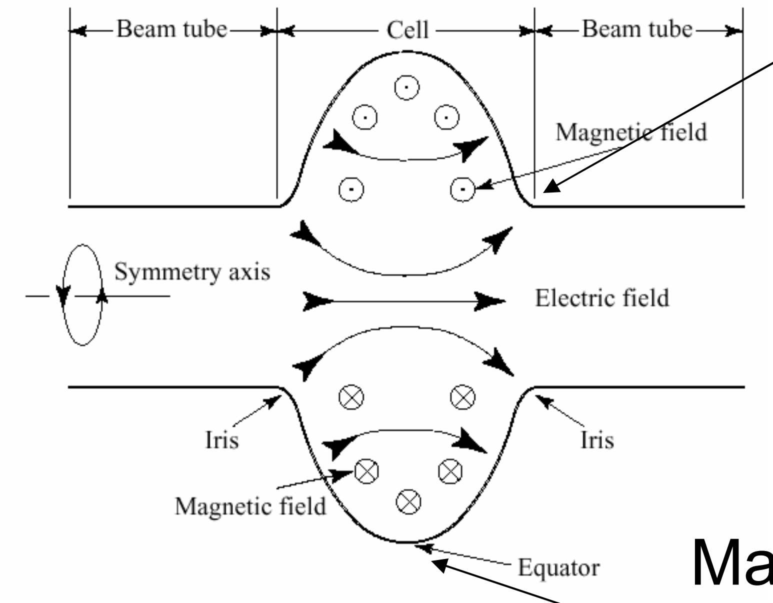


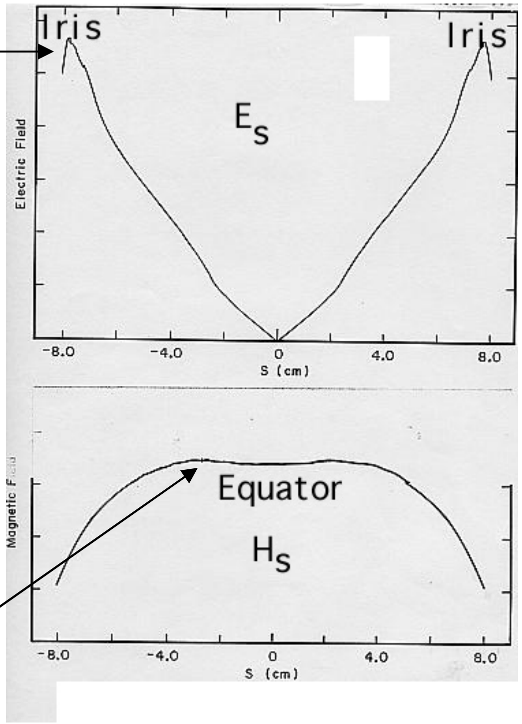
Real Cavities



Single Cell Cavities

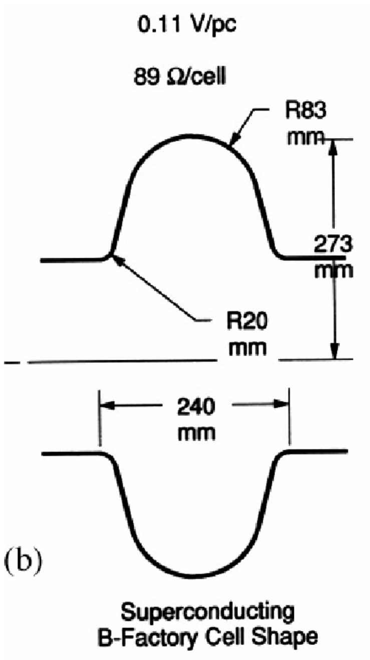
Electric field high at iris





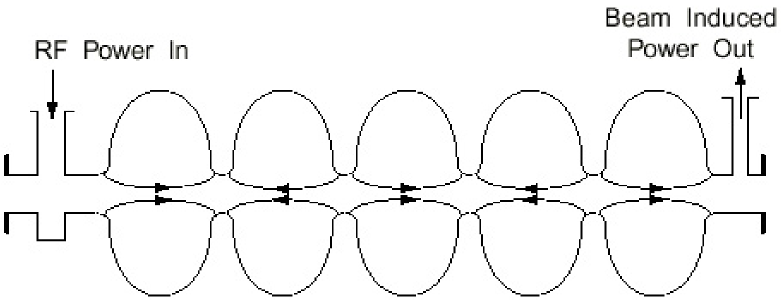
Magnetic field high at equator

Single Cell Cavities



Quantity	Cornell SC 500 MHz	Pillbox
G	270 ohmΩ	257 Ω
R_a/Q_0	88 ohm/cell	196 Ω/cell
E_{pk}/E_{acc}	2.5	1.6
H_{pk}/E_{acc}	52 Oe/MV/m	30.5 Oe/(MV/m)

Multi-Cell Cavities



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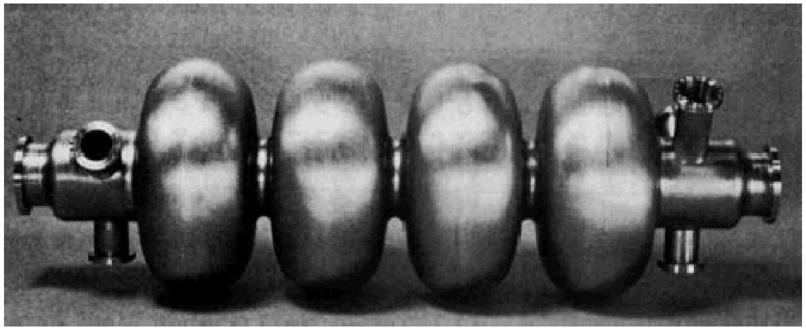
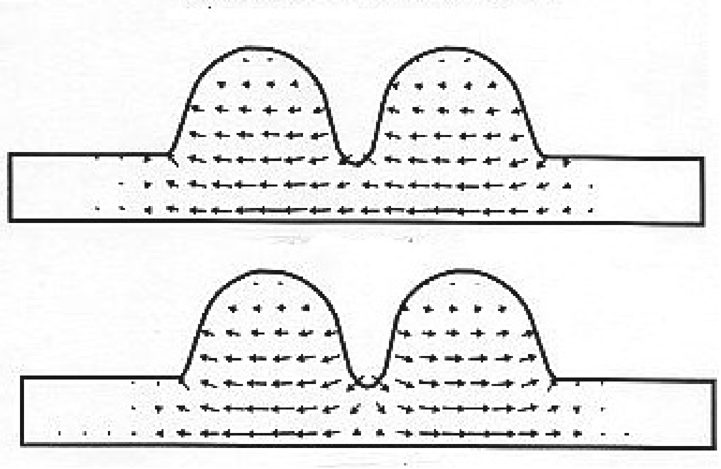
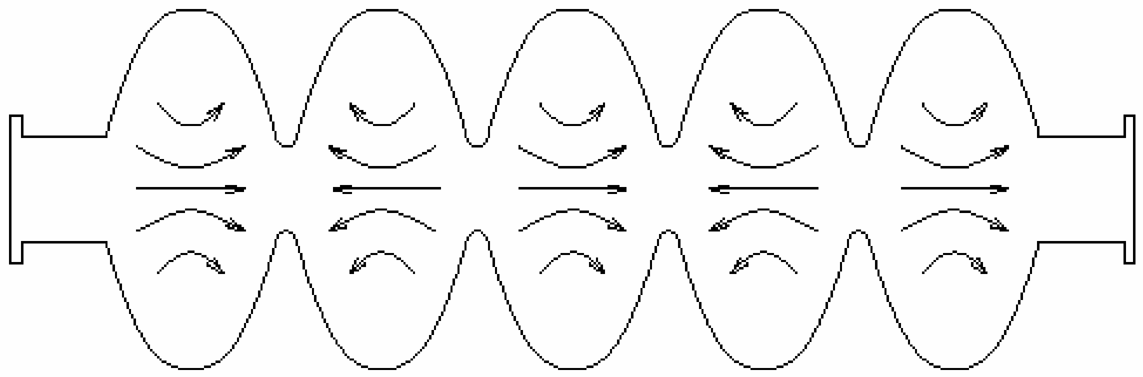


Multi-Cell Cavities

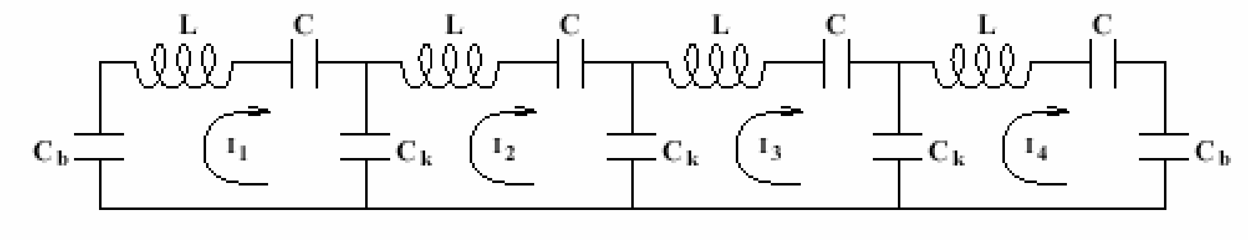
Modes of a 2 Cell Cavity





: Sketch of the electric field lines of the π-mode of a 5-cell

Multi-Cell Cavities



$$k = \frac{C}{C_k} \qquad C_b = C_k / 2$$

Mode frequencies:

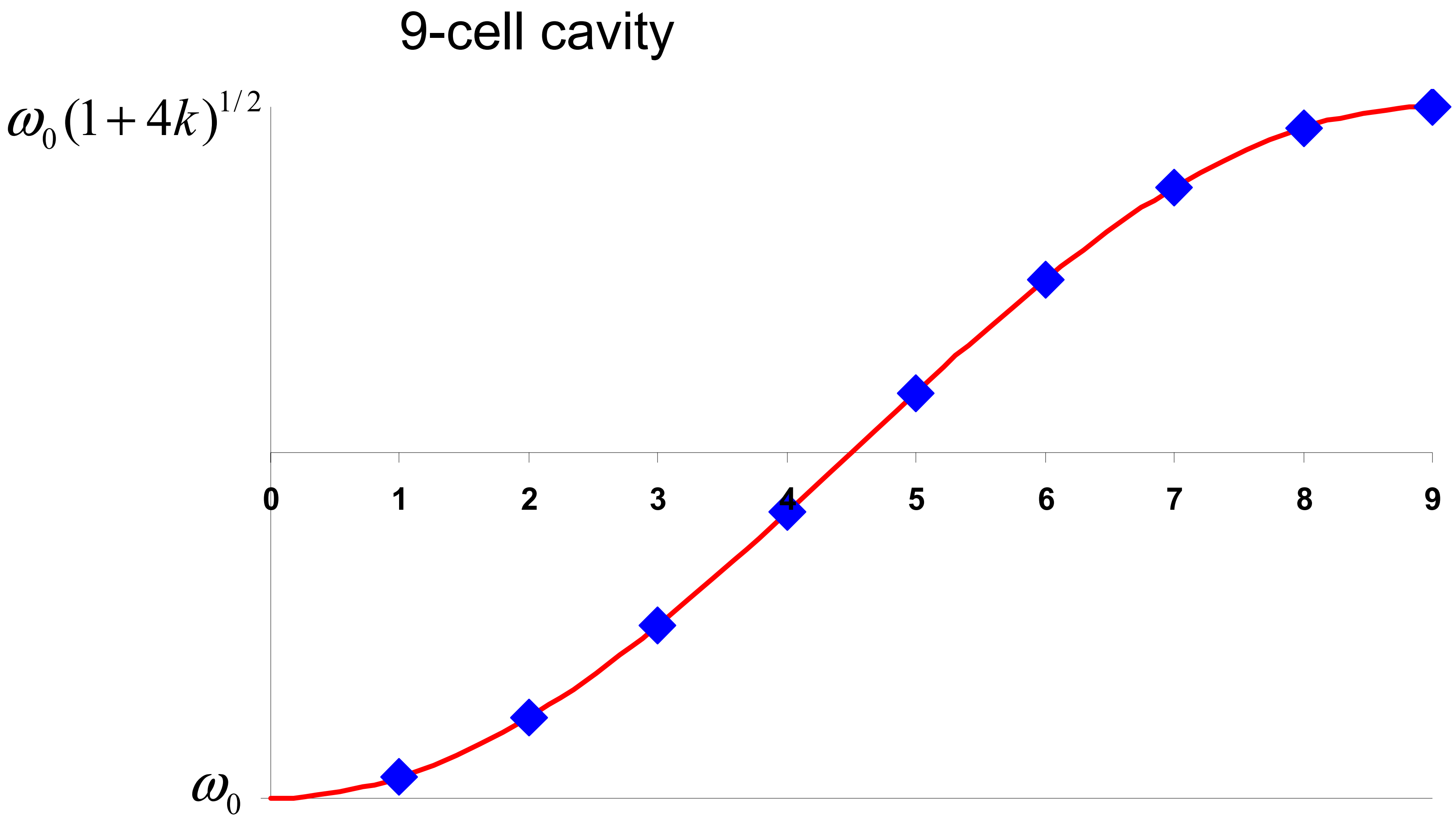
$$\frac{\omega_m^2}{\omega_0^2} = 1 + 2k\left(1 - \cos\frac{\pi m}{n}\right)$$

$$\frac{\omega_n - \omega_{n-1}}{\omega_0} \simeq k\left(1 - \cos\frac{\pi}{n}\right) \simeq \frac{k}{2}\left(\frac{\pi}{n}\right)^2$$

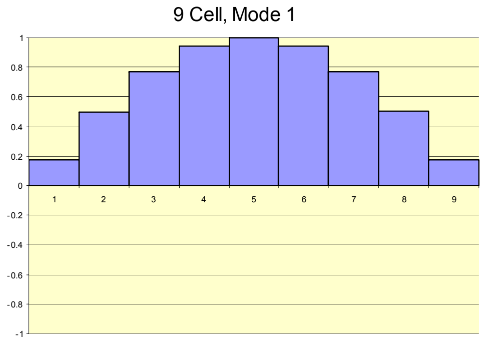
Voltages in cells:

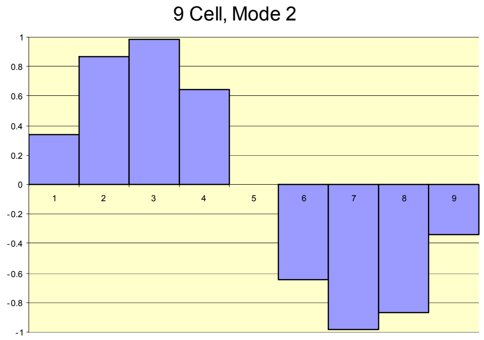
$$V_j^m = \sin\left(\pi m \frac{2j-1}{2n}\right)$$

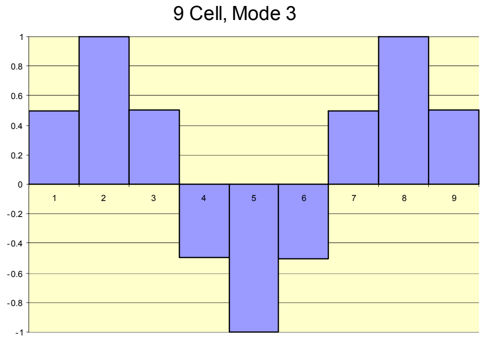
Pass-Band Modes Frequencies

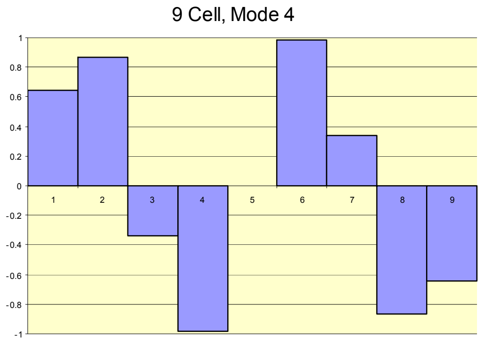


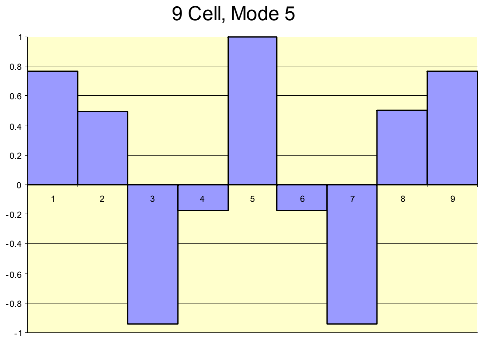
Cell Excitations in Pass-Band Modes

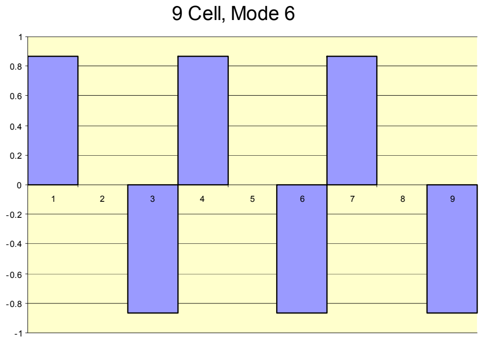

9 Cell, Mode 1

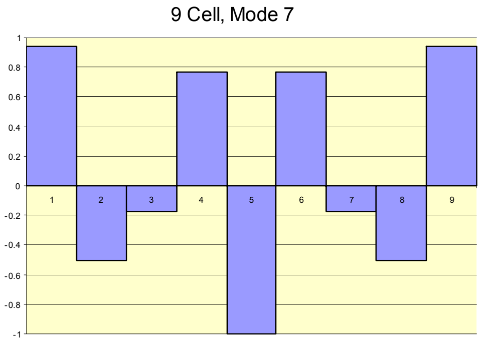

9 Cell, Mode 2

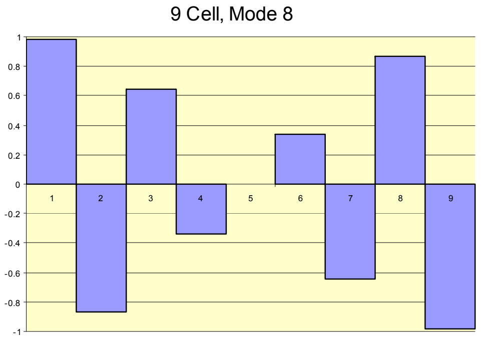

9 Cell, Mode 3

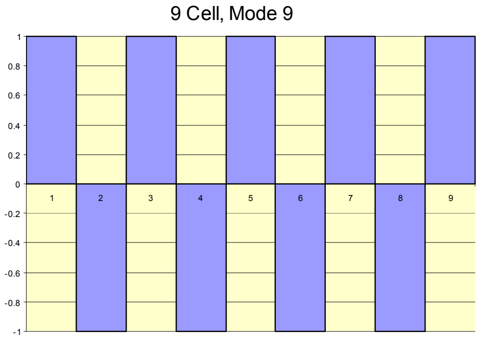

9 Cell, Mode 4


9 Cell, Mode 5


9 Cell, Mode 6


9 Cell, Mode 7


9 Cell, Mode 8


9 Cell, Mode 9

Field Flatness

Geometrical differences between cells causes a mixing of the eigenmodes

Sensitivity to mechanical deformation depends on mode spacing

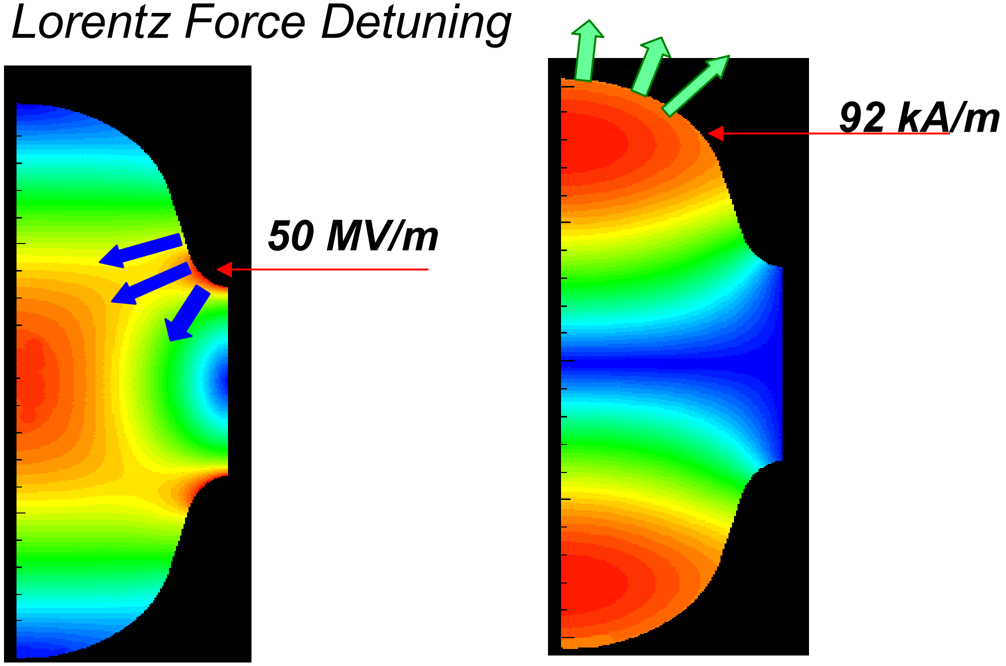
$$\frac{\omega_n - \omega_{n-1}}{\omega_0} \simeq k\left(1 - \cos\frac{\pi}{n}\right) \simeq \frac{k}{2}\left(\frac{\pi}{n}\right)^2$$

Mechanical Design

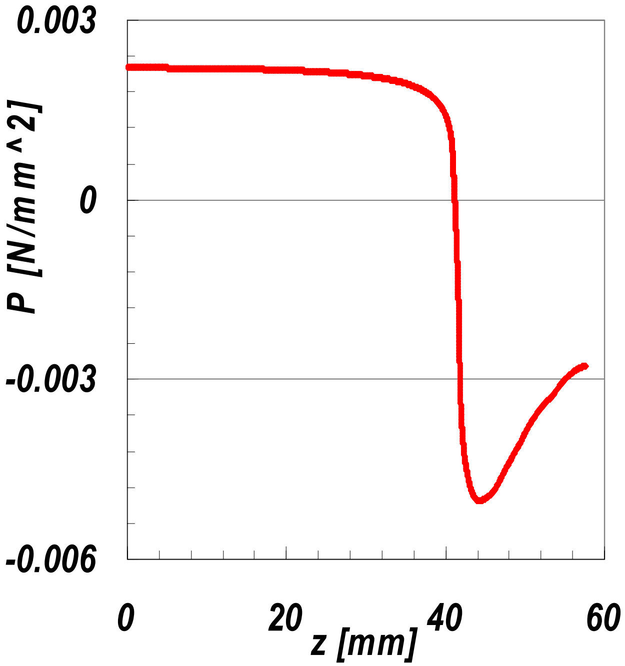
The mechanical design of a cavity follows its RF design:

- **Lorentz Force Detuning**
- **Mechanical Resonances**

Lorentz Force Detuning

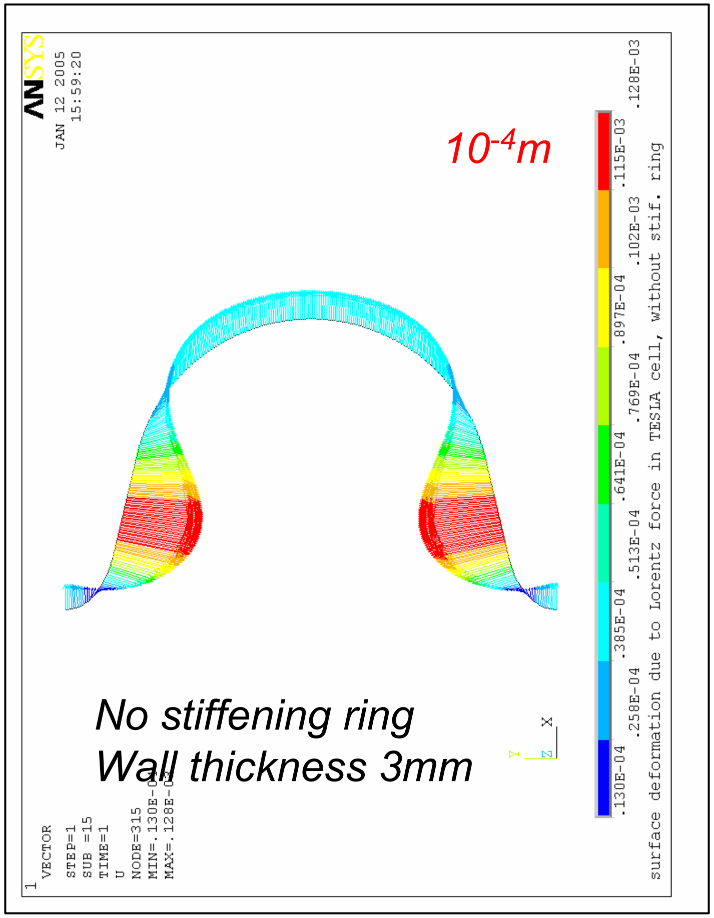


$$P = \frac{\mu_0 H_s^2 - \varepsilon_0 E_s^2}{4}$$



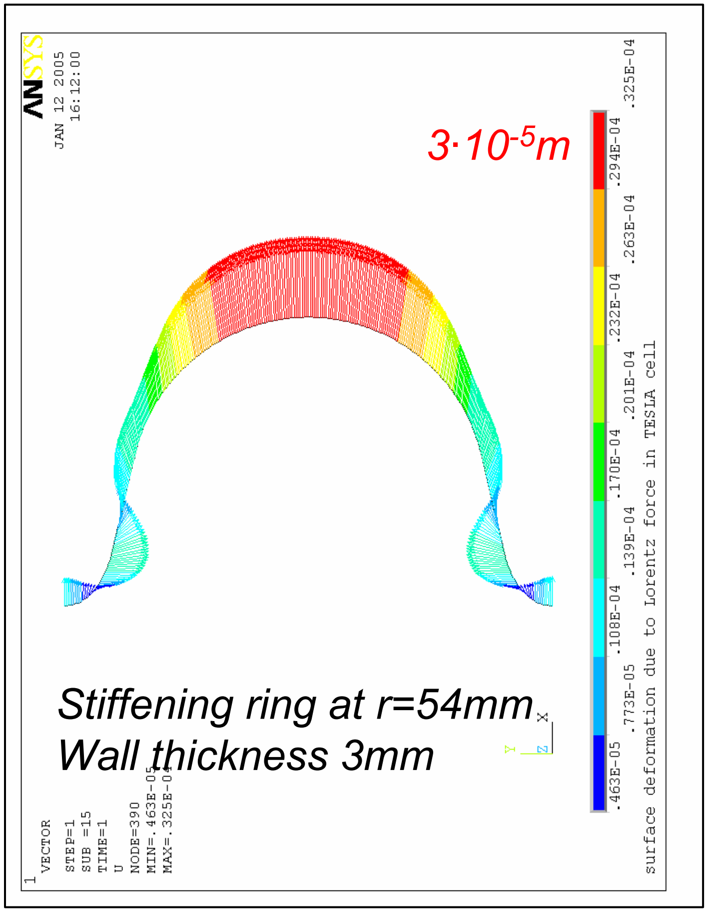
E and H at E_{acc} = 25 MV/m in TESLA inner-cup

Mechanical Design



$10^{-4}m$

No stiffening ring
Wall thickness 3mm



$3 \cdot 10^{-5}m$

Stiffening ring at r=54mm
Wall thickness 3mm

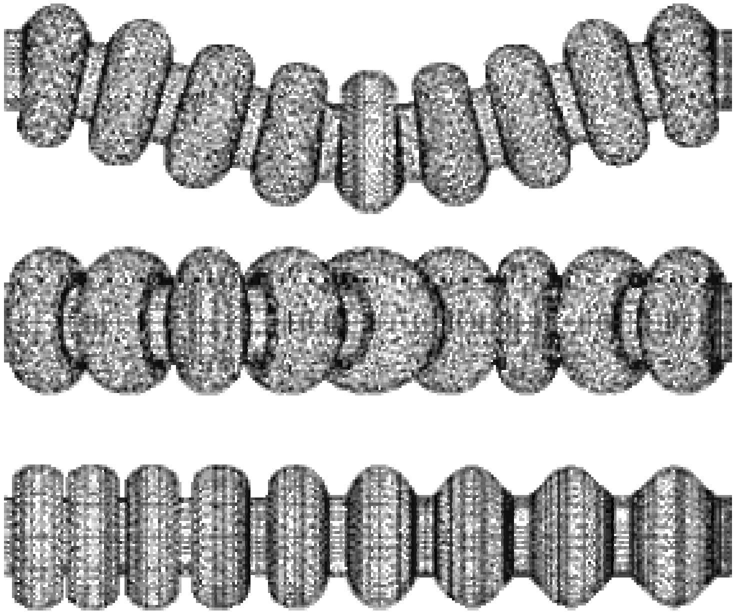
$k_L = -1\ Hz/(MV/m)^2$

Essential for the operation of a pulsed accelerator

$\Delta f = k_L(E_{acc})^2$

Mechanical Design

Mechanical Resonances of a multi-cell cavity



60 Hz

152 Hz

Transverse modes

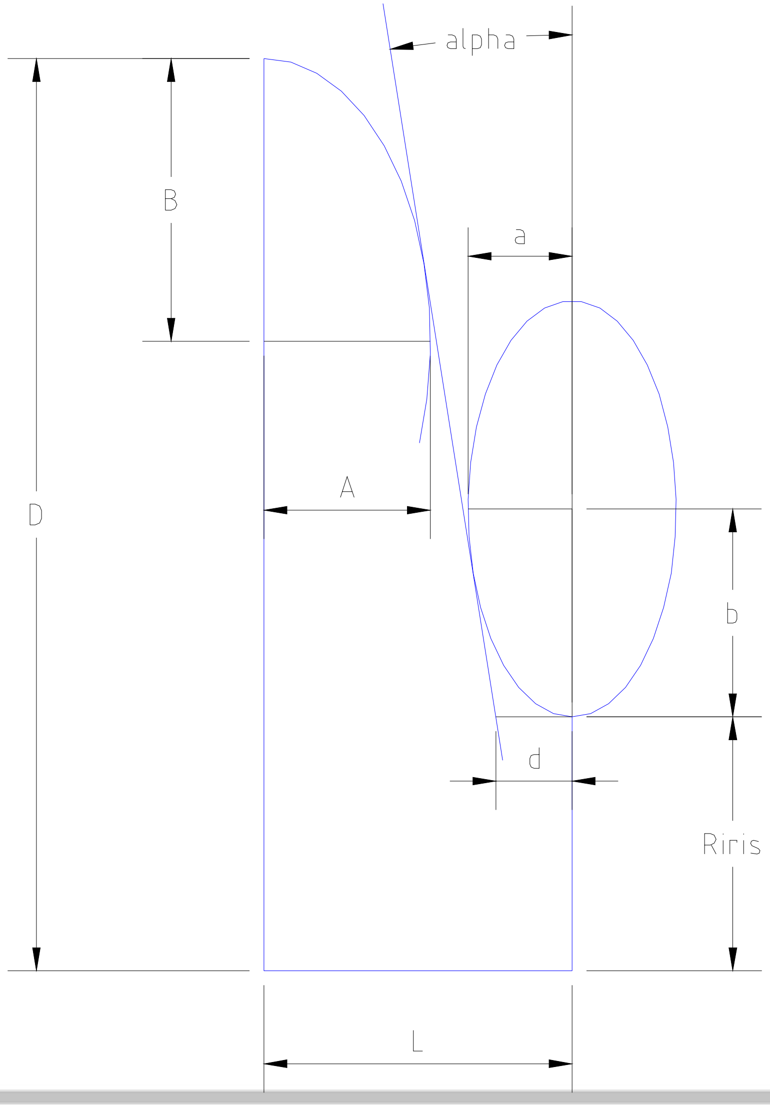
250 Hz *Longitudinal mode*

TESLA structure

The mechanical resonances modulate frequency of the accelerating mode.
Sources of their excitation: vacuum pumps, ground vibrations…

SNS Cell Shape Parametrization

- Full parametric model of the cavity in terms of 7 meaningful geometrical parameters:
  - ✓ Ellipse ratio at the equator (R=B/A)
    ruled by mechanics
  - ✓ Ellipse ratio at the iris (r=b/a)
    Epeak
  - ✓ Side wall inclination (a)
    and position (d)
    Epeak vs. Bpeak tradeoff and coupling k
  - ✓ Cavity iris radius Riris
    coupling k
  - ✓ Cavity Length L
    β
  - ✓ Cavity radius D
    used for frequency tuning
- Behavior of all e.m. and mechanical properties has been found as a function of the above parameters

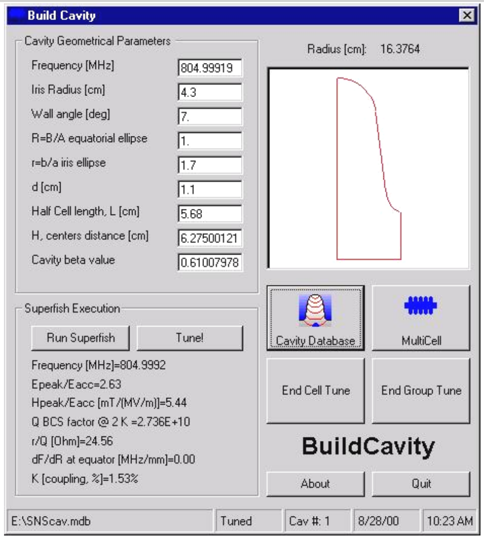


Tools used for the parametrization

Using of the parametric tool developed at *INFN Milano* for the analysis of the cavity shape on the electromagnetic parameters:

- All RF computations are handled by SUPERFISH
- Inner cell tuning is performed through the cell diameter, all the characteristic cell parameters stay constant: R, r, α, d, L, Riris
- End cell tuning is performed through the wall angle inclination, α, or distance, d.

  R, L and Riris are independently settable.
- Multicell cavity is then built to minimize the field unflatness, compute the effective β and the final cavity performances.
- A proper file to transfer the cavity geometry to ANSYS is then generated

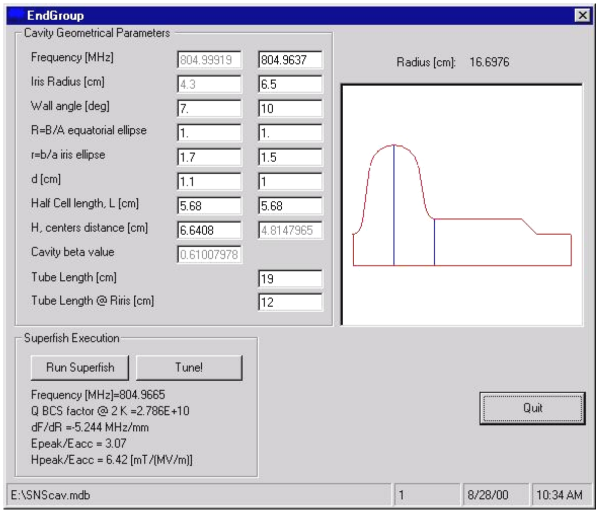


Radius [cm]: 16.3764

Frequency [MHz]	804.99919
Iris Radius [cm]	4.3
Wall angle [deg]	7.
R=B/A equatorial ellipse	1.
r=b/a iris ellipse	1.7
d [cm]	1.1
Half Cell length, L [cm]	5.68
H, centers distance [cm]	6.27500121
Cavity beta value	0.61007978

Frequency [MHz]=804.9992
Epeak/Eacc=2.63
Hpeak/Eacc [mT/(MV/m)]=5.44
Q BCS factor @ 2 K =2.736E+10
r/Q [Ohm]=24.56
dF/dR at equator [MHz/mm]=0.00
K [coupling, %]=1.53%

E:\SNScav.mdb | Tuned | Cav #: 1 | 8/28/00 | 10:23 AM



Radius [cm]: 16.6976

Frequency [MHz]	804.99919	804.9637
Iris Radius [cm]	4.3	6.5
Wall angle [deg]	7.	10
R=B/A equatorial ellipse	1.	1.
r=b/a iris ellipse	1.7	1.5
d [cm]	1.1	1
Half Cell length, L [cm]	5.68	5.68
H, centers distance [cm]	6.6408	4.8147965
Cavity beta value	0.61007978	
Tube Length [cm]		19
Tube Length @ Riris [cm]		12

Frequency [MHz]=804.9665
Q BCS factor @ 2 K =2.786E+10
dF/dR =-5.244 MHz/mm
Epeak/Eacc = 3.07
Hpeak/Eacc = 6.42 [mT/(MV/m)]

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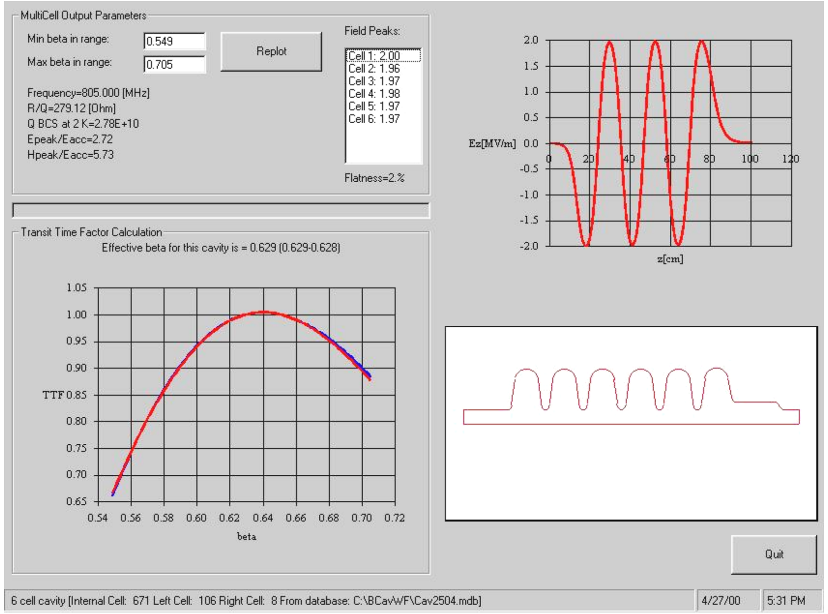


Min beta in range: 0.549
Max beta in range: 0.705

Frequency=805.000 [MHz]
R/Q=279.12 [Ohm]
Q BCS at 2 K=2.78E+10
Epeak/Eacc=2.72
Hpeak/Eacc=5.73

Cell 1: 2.00
Cell 2: 1.96
Cell 3: 1.97
Cell 4: 1.98
Cell 5: 1.97
Cell 6: 1.97

Flatness=2.%

Effective beta for this cavity is = 0.629 (0.629-0.628)

6 cell cavity [Internal Cell: 671 Left Cell: 106 Right Cell: 8 From database: C:\BCavWF\Cav2504.mdb] | 4/27/00 | 5:31 PM

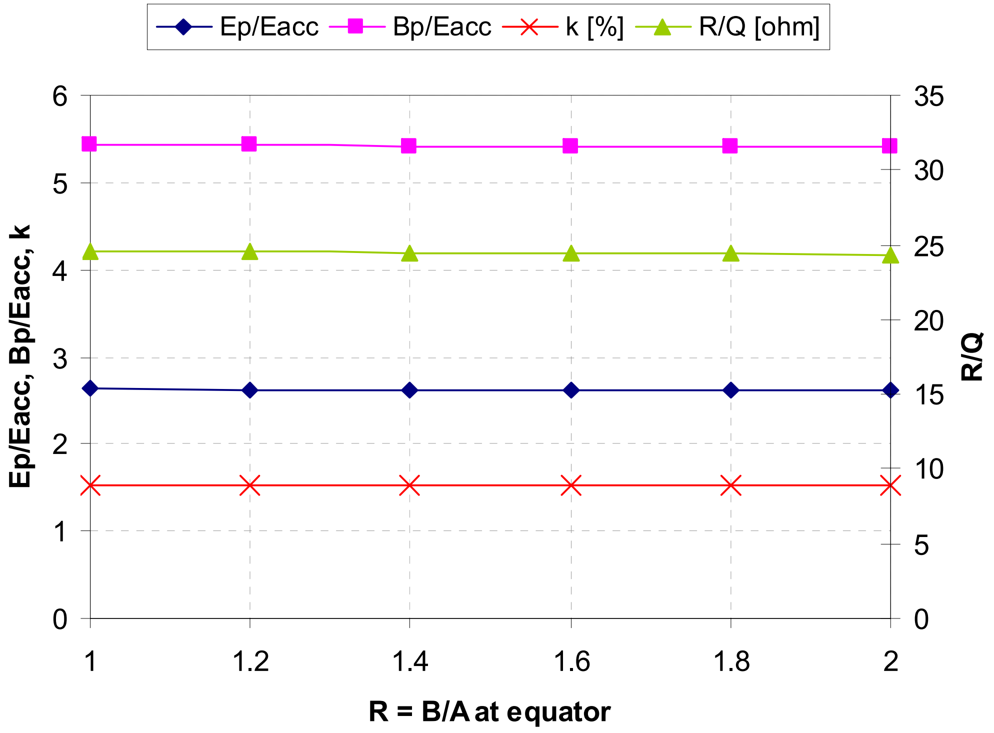
Inner cell data

L = 56.8 mm
R = 1
r = 1.7
α = 7°
d = 11 mm
R_{iris} = 43 mm

R: “mechanical” parameter

- The equator aspect ratio (R) is a free parameter for what concerns the π-mode e.m. design but the cavity mechanical parameters are greatly affected by the equator shape:
- R>1 allows better stress distribution in the unstiffened cavity but a bigger Lorentz force coefficient
- The cell tuning strategy doesn’t affect the cell’s performances (e.m. and mechanical)

Chosen R = 1



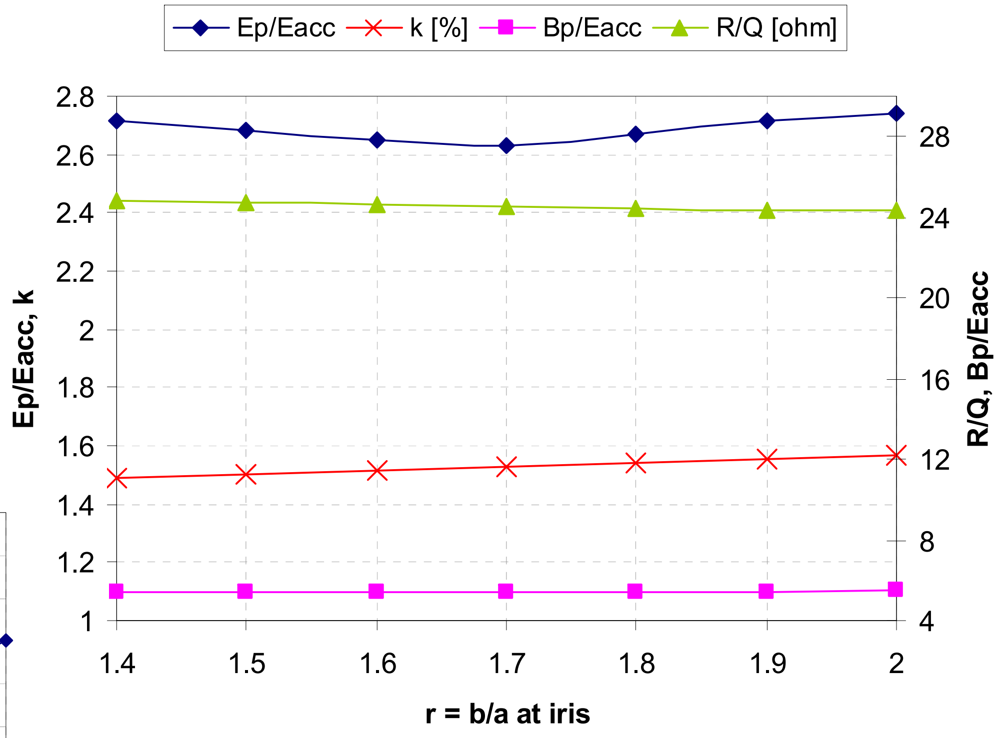
Reference data

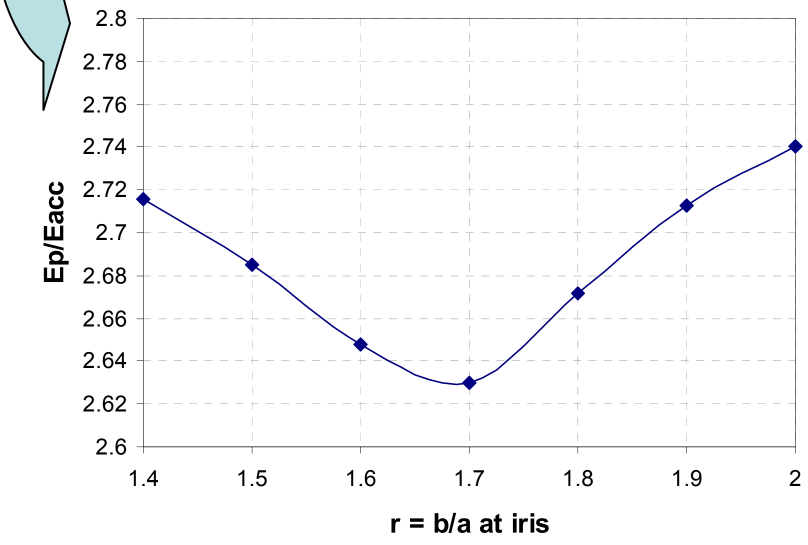
L = 56.8 mm r = 1.7 α = 7°

d = 11 mm R_{iris} = 43 mm

Optimal value for r

- The iris ellipse aspect ratio has always an optimal value that minimize the peak surface E field
- All the other cavity parameters (e.m. and mechanical) are unchanged



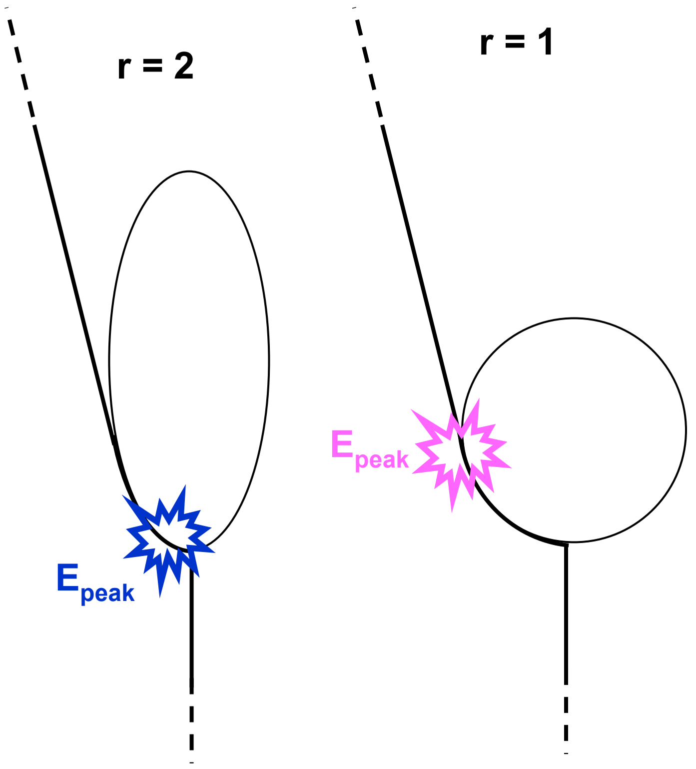


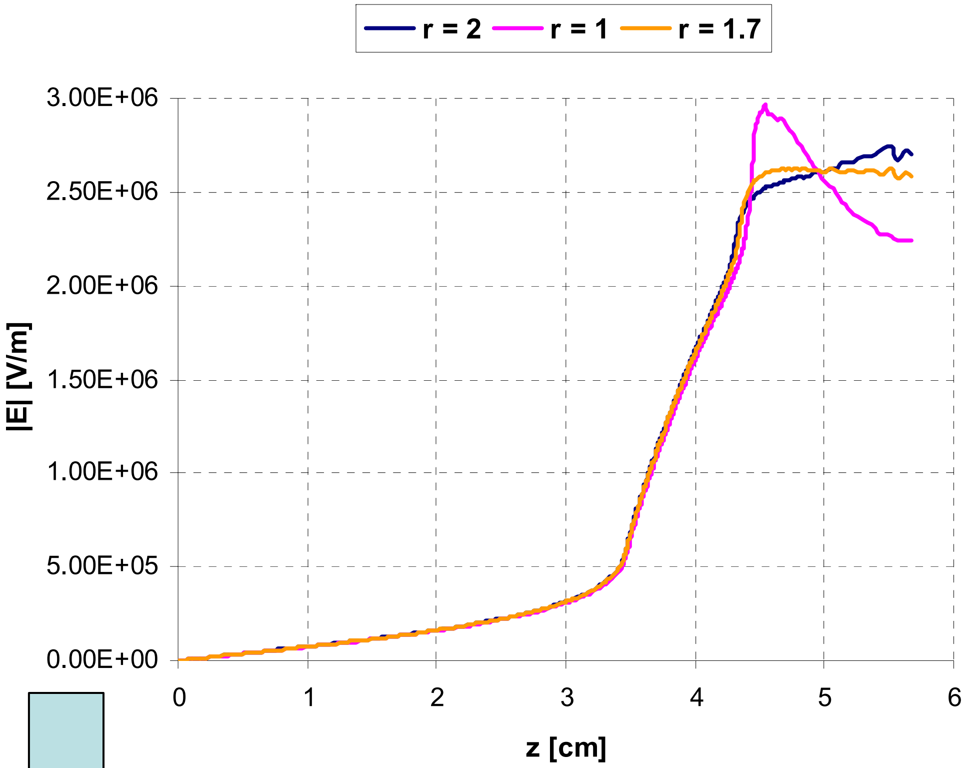
Reference data

L = 56.8 mm   R= 1 α = 7°

d = 11 mm R_{iris} = 43 mm

Optimal value for r

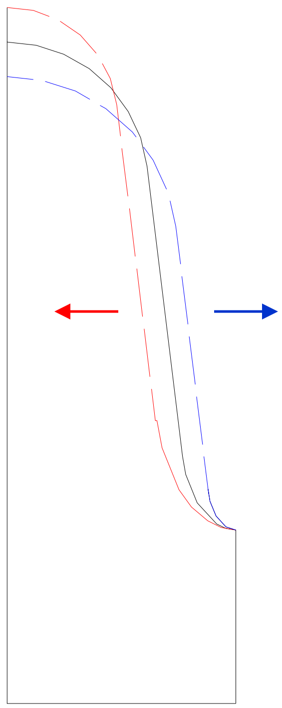




Optimization of r means best field distribution on the cell’s nose

Chosen r = 1.7

Influence of d @ constant R_{iris}



- d can be used to balance the electric and magnetic volumes of the cavity
  - The value of r is always optimal.
  - The cell to cell coupling changes, if R_{iris} is kept constant

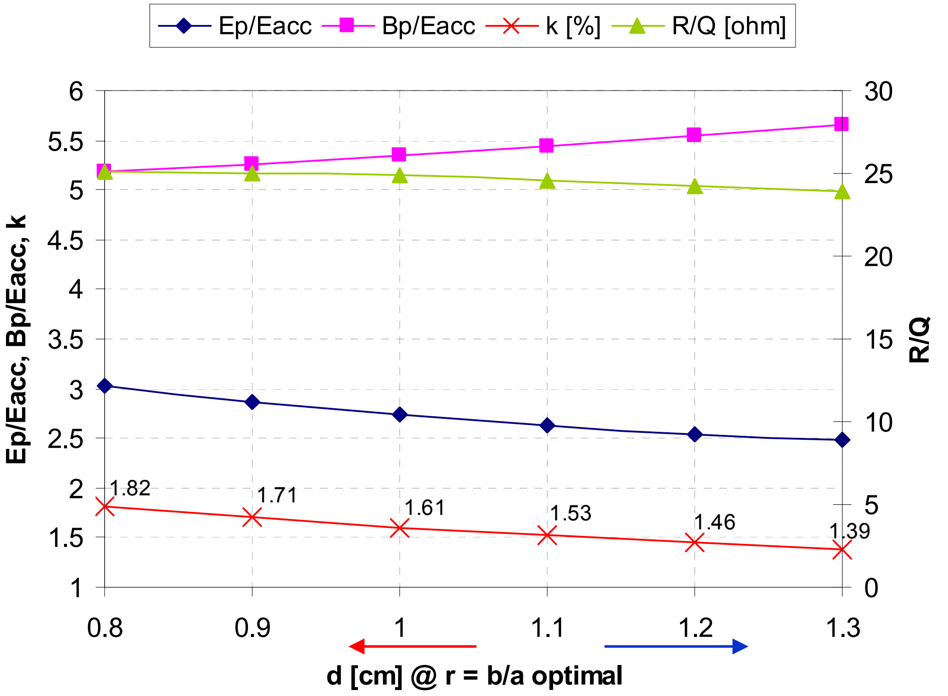
Reference data

L = 56.8 mm

R = 1 α = 7°

r = optimal

R_{iris} = 43 mm



- Better mechanical performances are reached with decreasing d

Influence of d @ constant k

- If we want to keep a constant cell to cell coupling we have to adjust Riris

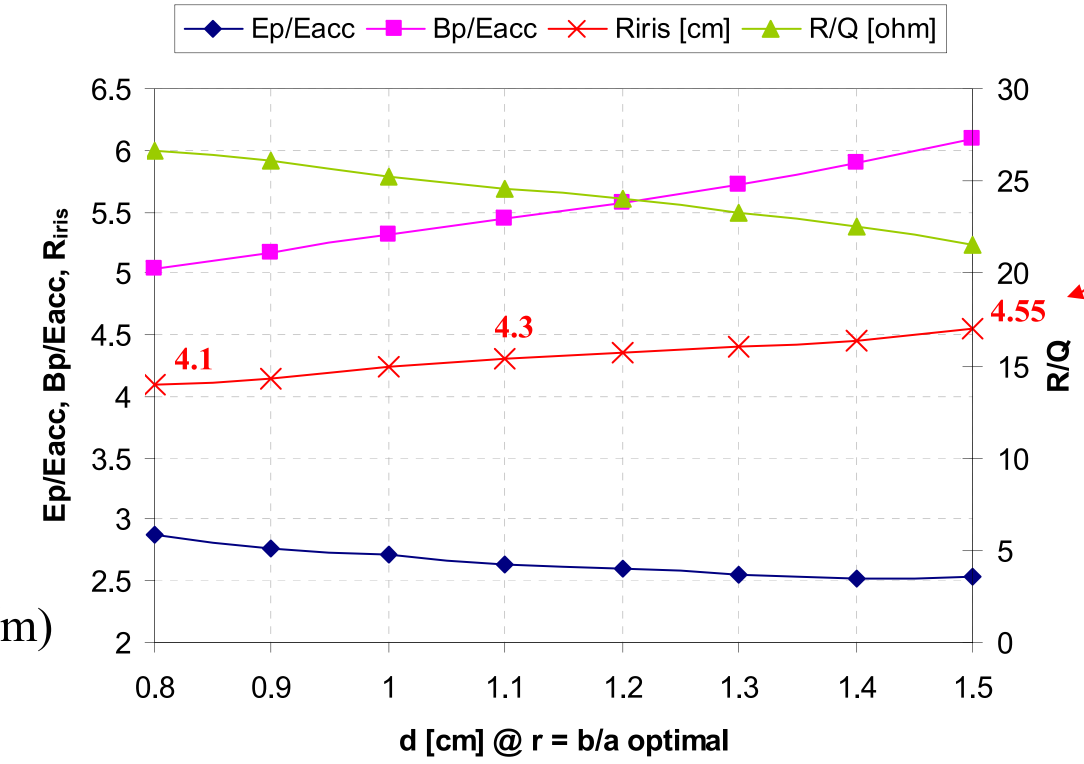
Reference data

L = 56.8 mm
R = 1

$\alpha = 7°$
r = optimal
k = 1.5 %

Chosen d=11 mm

$B_{peak}/E_{peak} = 2.07\ \mathrm{mT/(MV/m)}$

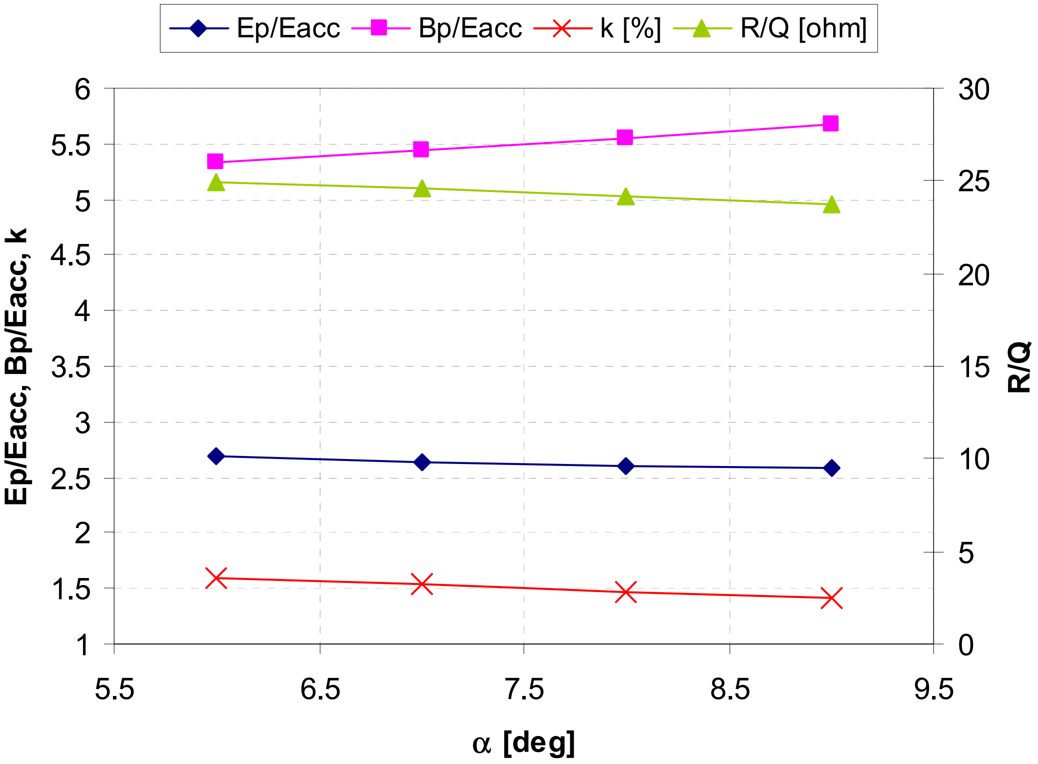


Dependence on α

The wall angle α slightly affects all the e.m. parameters, but has a strong effect on the mechanical performances:

- **Lower values are preferred for Lorentz force detuning**
- **Too small α could be critical for chemistry and cleaning**

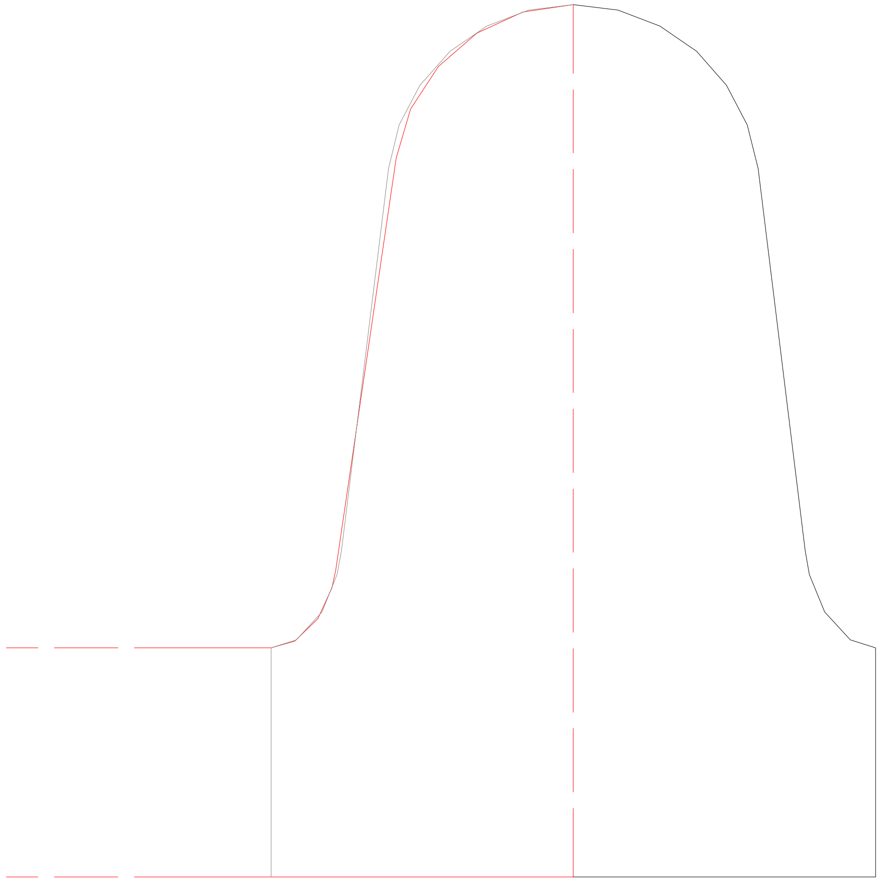
Chosen α = 7 deg



Reference data

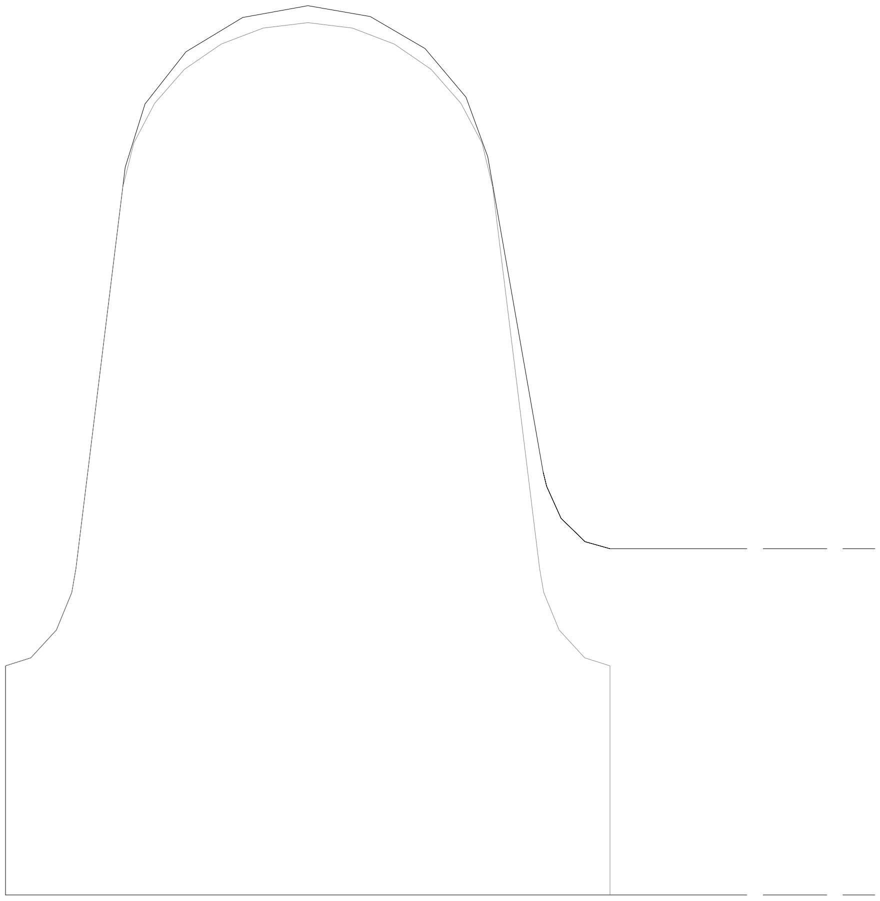
L = 56.8 mm R = 1
d =11 mm r = 1.7
R_{iris} = 43 mm

End cell tune



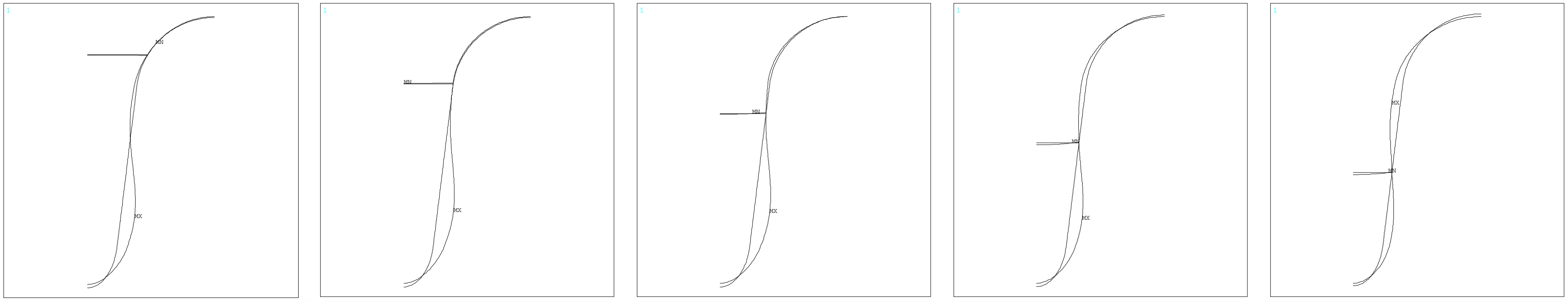
- d set 1 mm lower than the in-cell
- optimization of r = b/a at iris
- Slater compensation (decrease of the magnetic volume) of the cut-off tube and d reduction (↓f), increasing the wall angle α. This gives also the necessary stiffening to the end cell
- the frequency of end cell + tube is about 50 kHz lower than the in-cell's due to the asimmetry

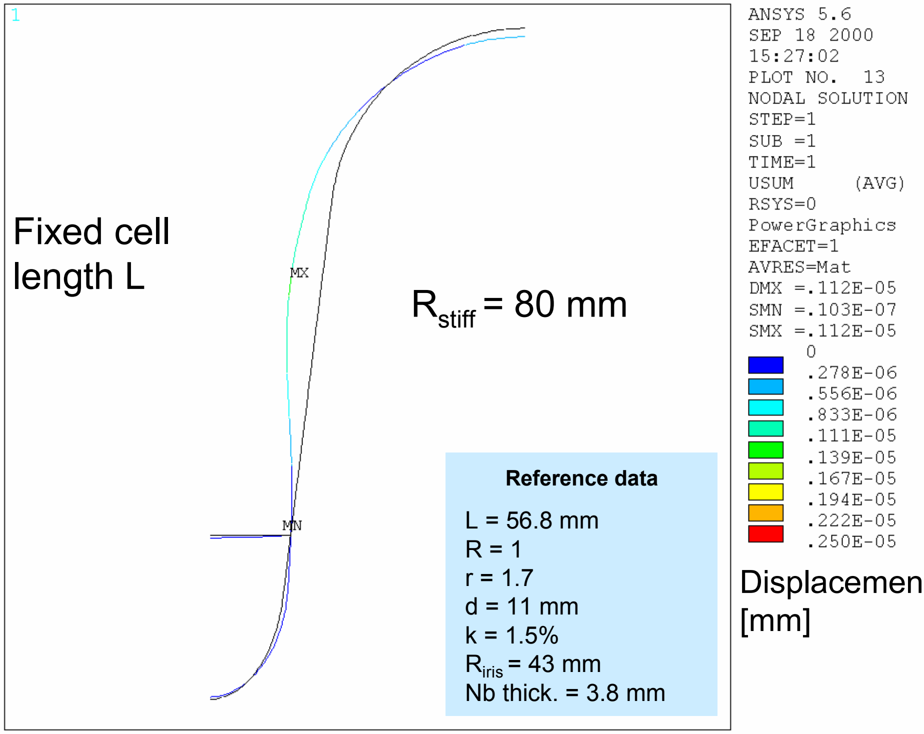
End cell @ FPC side tune



- R_{iris} set to 65 mm to have enough field at the power coupler antenna
- d set 1 mm lower than the in-cell
- optimization of r = b/a at iris
- α set to 10 deg to have the necessary stiffening
- Slater compensation (increase of the magnetic volume) of the cut-off tube (↓f), d reduction (↓f), α and R_{iris} increase (↑f) by increasing the equator radius ⇨ 4 dies
- the frequency of end cell + tube is about 40 kHz lower than the in-cell's due to the asimmetry

Optimal stiffening ring position





Fixed cell length L

R_{stiff} = 80 mm

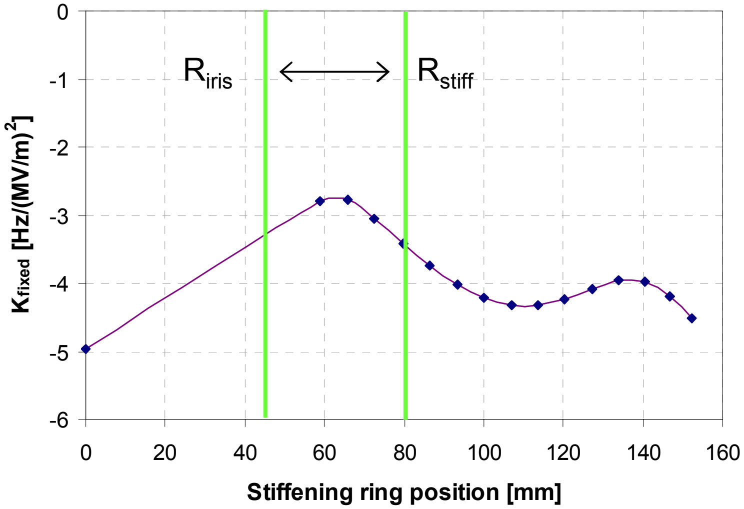
Reference data

L = 56.8 mm
R = 1
r = 1.7
d = 11 mm
k = 1.5%
R_{iris} = 43 mm
Nb thick. = 3.8 mm

```
ANSYS 5.6
SEP 18 2000
15:27:02
PLOT NO.   13
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
USUM      (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =.112E-05
SMN =.103E-07
SMX =.112E-05
      0
🟦    .278E-06
🟦    .556E-06
🟦    .833E-06
🟩    .111E-05
🟩    .139E-05
🟩    .167E-05
🟨    .194E-05
🟧    .222E-05
🟥    .250E-05
```

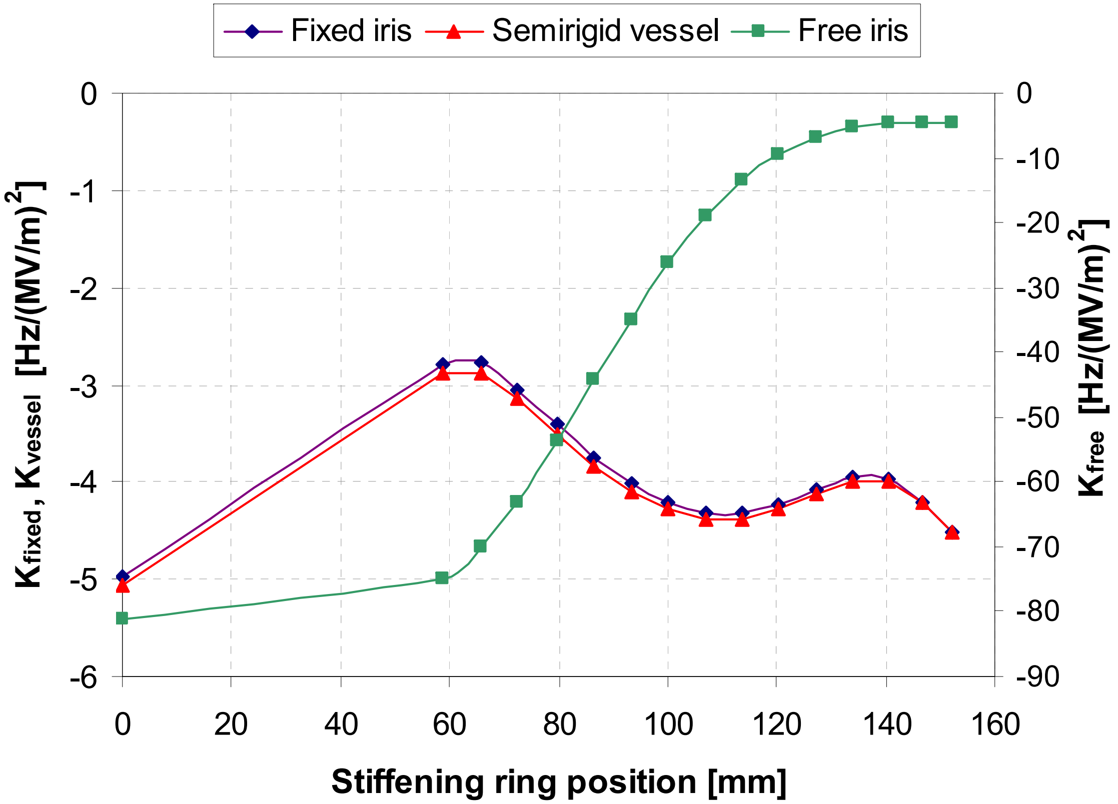
Displacements [mm]

The Lorentz forces coefficients for 15 different stiffening ring positions are evaluated automatically with ANSYS, preparing the geometry and reading the fields from the SFO output from SUPERFISH



K_L for different boundary conditions

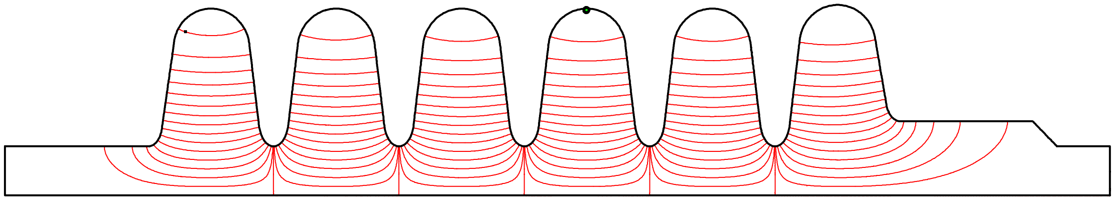
- The estimate for K_L strongly depends on the cell boundaries. We compute it for 3 different cases:
  - Fixed cell length
  - Free cell length
  - Helium Vessel/Tuning System (= 3 tubes with diameter 30 mm and thickness 2 mm)



β_g = 0.61 Cavity for SNS

Effective β that matches the TTF curve = 0.630

E_p/E_{acc}	2.72 (2.63 inner cell)
B_p/E_{acc} [mT/(MV/m)]	5.73 (5.44 inner cell)
R/Q [Ω]	279
G [Ω]	214
k [%]	1.53
Q_{BCS} @ 2 K [10^9]	27.8
Frequency [MHz]	805.000
Field Flatness [%]	2



KL70 = -2.9 [Hz/(MV/m)^2] KL80 = -3.4 [Hz/(MV/m)^2]

Nb thickness = 3.8 mm-

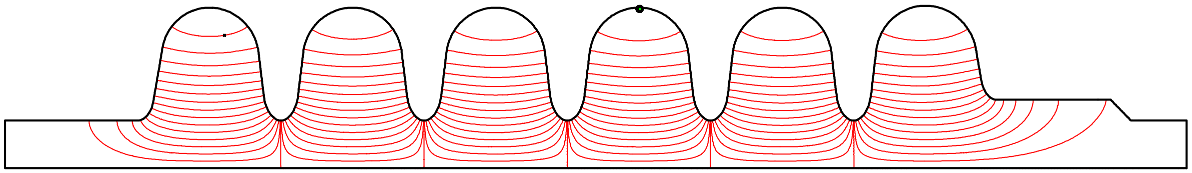
Geometrical Parameters

	Inner cell	End Cell Left	End Group (coupler) Left	End Group (coupler) Right
L [cm]	5.68	5.68	5.68	
R_{iris} [cm]	4.3	4.3	4.3	6.5
D [cm]	16.376	16.376	16.698	
d [cm]	1.1	1.0	1.1	1.0
r	1.7	1.5	1.7	1.5
R	1.0	1.0	1.0	
α [deg]	7.0	8.36	7.0	10.0

β_g = 0.81 Cavity for SNS

Effective β that matches the TTF curve = 0.832

E_p/E_{acc}	2.19 (2.14 inner cell)
B_p/E_{acc} [mT/(MV/m)]	4.72 (4.58 inner cell)
R/Q [Ω]	484.8
G [Ω]	233
k [%]	1.52
Q_{BCS} @ 2 K [10^9]	36.2
Frequency [MHz]	805.004
Field Flatness [%]	1.1



KL70 = -0.7 [Hz/(MV/m)^2] KL80 = -0.8 [Hz/(MV/m)^2]

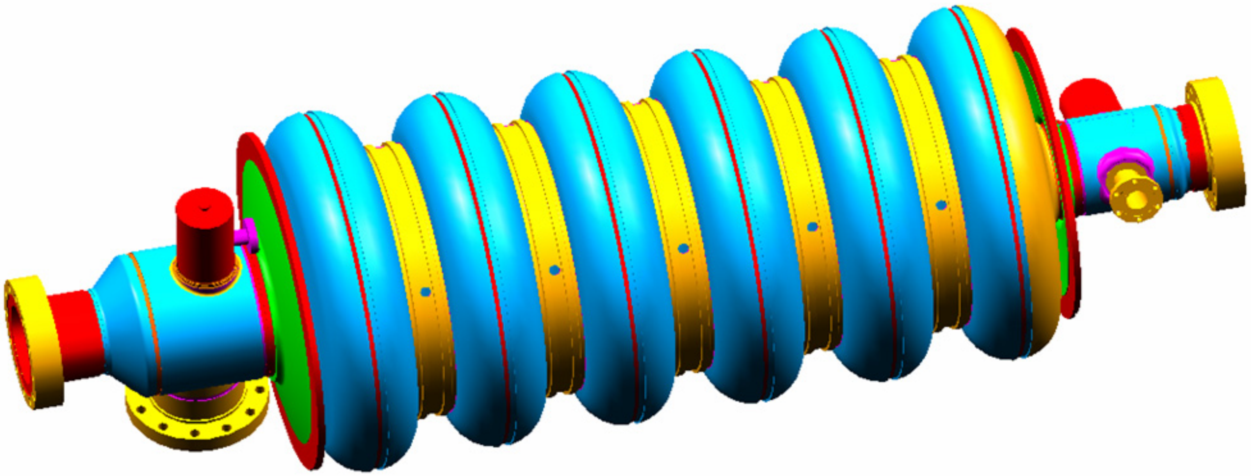
Nb thickness = 3.8 mm-

Geometrical Parameters

	Inner cell	**End Cell Left**	**End Group (coupler)**	
			Left	**Right**
L [cm]	**7.55**	**7.55**	**7.55**	
R_{iris} [cm]	**4.88**	**4.88**	**4.88**	**7.0**
D [cm]	**16.415**	**16.415**	**16.611**	
d [cm]	**1.5**	**1.3**	**1.5**	**1.3**
r	**1.8**	**1.6**	**1.8**	**1.6**
R	**1.0**	**1.0**	**1.0**	
α [deg]	**7.0**	**10.072**	**7.0**	**10.0**

Stress and Modal Analysis

- Nominal Medium Beta Cavity



SNS Cavity Modal Analysis

Medium Beta Cavity

End Condition	Load (atm)	127 mm Rings (Hz)	70 mm Rings (Hz)	No Rings (Hz)
Fixed-Guided	-	85	48	38
Fixed-Fixed	-	126 (*204)	57 (*59)	48 (*42)
Fixed-Fixed Mid Supt	-	149 (*220)	95 (~*108)	88
Compressed 0.4mm	1.65	125	-	46
Compressed 1.25 mm	1.65	124	-	46

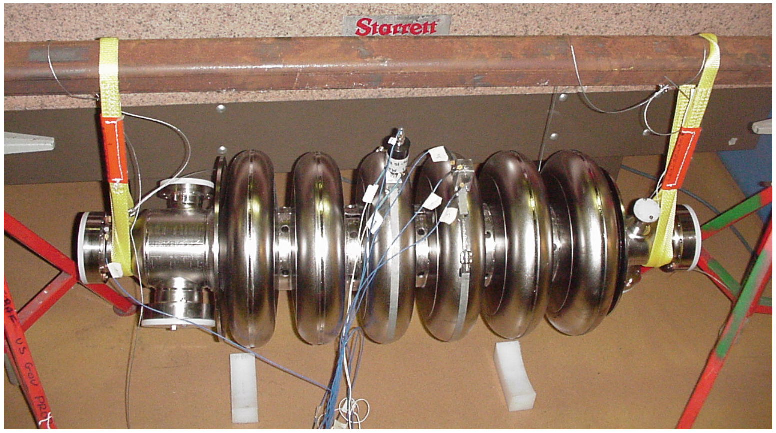
(*D. Schrage, LANL)
(~ Beta = 0.76)

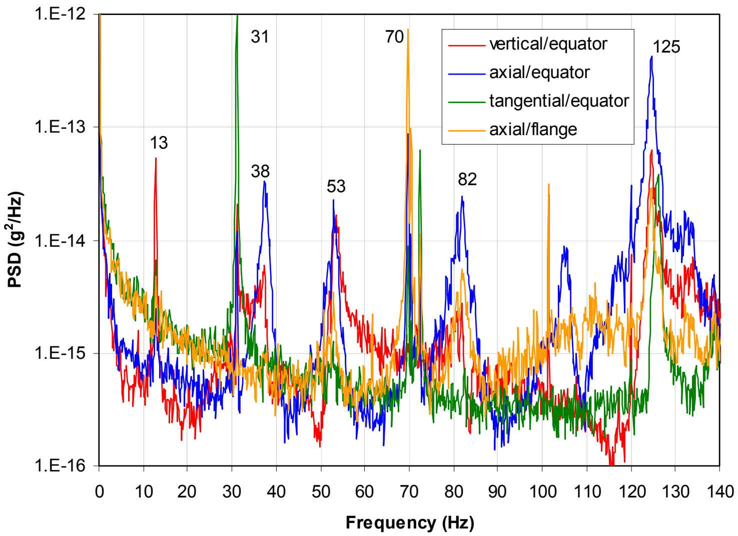
SNS Cavity Modal Analysis

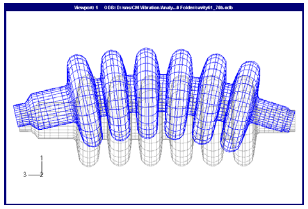
High Beta Cavity

End Condition	Load (atm)	127 mm Rings (Hz)	70 mm Rings (Hz)	No Rings (Hz)
Fixed-Fixed	-	120	-	46
Fixed-Guided	-	107	-	34
Compressed 0.4mm	1.65	120	-	44
Compressed 1.25 mm	1.65	119	-	44

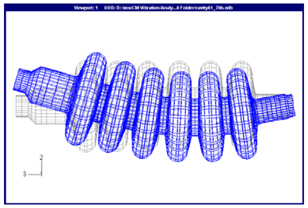
Mode Analysis, Beta = 0.81



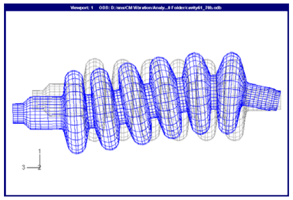




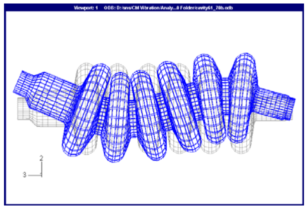
Mode 1 – 14 Hz



Mode 2 – 26 Hz



Mode 3 – 40 Hz



Mode 5 – 72 Hz

	Natural Frequency (Hz)	
Mode	Test Data	FE Analysis
1	13	14
2	31	26
3	38	40
4	53	48
5	70	72
6	82	83
7	125	124

SNS Cavity Mechanical Design Requirements

- Minimize/prevent microphonics
- Withstand loss of vacuum accident up to 5 atm
- Withstand cool down at 1.65 atm
- Adhere to intent of ASME B&P Code
  - Allowable Stress (Sm) = 2/3 Yield Stress
  - Primary Membrane Stress (Pm) <= (Sm)
  - Pm + Bending <= 1.5*Sm
  - Pm + Bending + Secondary Stress <= 3*Sm
    - Allowable Stresses
      - **Warm Niobium = 4,667 psi**
      - **Cold Niobium = 53,333 psi**

Medium Beta Stress Analysis

SNS Medium Beta Cavity Wall Stresses				
Compression (mm/end)	Loads (atm)	127 mm Stiffening Ring Max Stress (psi)	70 mm Stiffening Ring Max Stress (psi)	No Stiffening Ring Max Stress (psi)
0.2	1.65	-	-	-
0.4	1.65	3,960	-	4,310
0.5	1.65	4,610	-	4,550
0.75	1.65	7,500	-	4,670
1.25	1.65	17,500	5,730 (1.8 atm)	5,000
0.75	5	11,200	-	12,900
1.25	5	14,300	10,100	47,100

High Beta Stress Analysis

SNS High Beta Cavity Wall Stresses

Compression (mm/end)	Loads (atm)	127 mm Stiffening Ring Max Stress (psi)	70 mm Stiffening Ring Max Stress (psi)	No Stiffening Ring Max Stress (psi)
0.2	1.65	3,040	-	-
0.4	1.65	6,350	-	3,140
0.5	1.65	8,070	-	3,350
0.75	1.65	12,500	-	3,940
1.25	1.65	21,400	-	5,830
0.75	5	11,500	-	9,130
1.25	5	14,300	-	9,590